Reg.No.: | | | | | | | | | | | | |

VIVEKANANDHA COLLEGE OF ENGINEERING FOR WOMEN

[AUTONOMOUS INSTITUTION AFFILIATED TO ANNA UNIVERSITY, CHENNAI]

Elayampalayam – 637 205, Tiruchengode, Namakkal Dt., Tamil Nadu.

Question Paper Code: 20005

B.E. / B.Tech. DEGREE END-SEMESTER EXAMINATIONS – NOV. / DEC. 2024

Third Semester

Computer Science and Engineering

U23MA304 – DISCRETE MATHEMATICS

(Common to IT & CST)

(Regulation 2023)

Time : Three Hours | Maximum : 100 Marks

Answer ALL the questions

Knowledge Levels (KL)	K1 – Remembering	K3 – Applying	K5 - Evaluating
	K2 – Understanding	K4 – Analyzing	K6 - Creating

PART – A

(10 x 2 = 20 Marks)

Q.No.	Questions	Marks	KL	CO
1.	Construct the truth table for the compound proposition $(p \to q) \to (q \to p)$.	2	K2	CO1
2.	Define disjunctive and conjunctive normal form of a statement.	2	K1	CO1
3.	Let $E = \{-1,0,1,2\}$ denote the universe of discourse. If $P(x, y) = x + y + 1$, find the truth value of $(\forall x)(\exists y)P(x, y)$.	2	K2	CO2
4.	What are free and bound variables in predicate logic?	2	K1	CO2
5.	R is a relation from $A = \{1, 2, 3\}$ to $B = \{1, 2\}$ such that aRb if $a > b$. Write down the matrix representation of R.	2	K1	CO3
6.	Let $A = \{a, b, c\}$ and $\rho(A)$ be its power set, draw a Hasse diagram of $\langle \rho(A), \subseteq \rangle$.	2	K3	CO3
7.	If $f, g: R \to R$ are defined by $f(x) = 2x + 5$ and $g(x) = \frac{1}{2}(x - 5)$, show that f and g are inverse of each other.	2	K2	CO4
8.	If $f_1(a, b) = a + b, f_2(a, b) = a^2 + ab + b^2$ and $g(a, b) = ab$, find the composition of g with f_1 and f_2.	2	K3	CO4

Q.No.	Questions	Marks	KL	CO
9.	Prove that if G is abelian group, then for all $a, b \in G, (a * b)^2 = a^2 * b^2$	2	K1	CO5
10.	Find the left cosets of $\{[0], [3]\}$ in the addition modulo group $(Z_6, +_6)$.	2	K1	CO5

PART – B

(5 x 16 = 80 Marks)

Q.No.			Questions	Marks	KL	CO
11.	a)	i	Without using truth table Show that $q \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$ is a tautology.	8	K3	CO1
		ii	Without constructing the truth tables, obtain the principle disjunctive normal form of $(\neg p \rightarrow r) \wedge (q \leftrightarrow r)$.	8	K3	CO1
			(OR)			
	b)	i	Prove the implications without using truth tables $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \Rightarrow r$.	8	K3	CO1
		ii	Obtain the principal disjunctive normal forms and principal conjunctive normal form of $(p \rightarrow (q \wedge r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$.	8	K3	CO1
12.	a)	i	Show that the premise "one student in this class knows how to write programs in JAVA " and everyone who knows how to write programs in "JAVA can get high-paying Job " imply the conclusion "someone in this class can get high paying job".	8	K3	CO2
		ii	Show that $\forall x(P(x) \vee Q(x)) \Longrightarrow \forall x P(x) \vee \exists x Q(x)$ by indirect method.	8	K 3	CO2
			(OR)			
	b)	i	Show that the conclusion $\forall x(P(x) \rightarrow \neg Q(x))$ follows from the premises $\exists x(P(x) \wedge Q(x)) \rightarrow \forall y(R(y) \rightarrow S(y))$ and $\exists y\,(R(y) \wedge \neg S(y))$.	8	K 3	CO2
		ii	Establish the following logical equivalences, where A is a proposition not involving any quantifiers. $(\forall x P(x)) \wedge A \equiv \forall x(P(x) \wedge A)$ and $(\exists x P(x)) \wedge A \equiv \exists x(P(x) \wedge A)$.	8	K 3	CO2
13.	a)	i	The equivalence relation R on the set $A = \{1, 2, 3, 4, 5, 6\}$ is given below, $R = \left\{\begin{matrix}(1,1),(1,2),(2,1),(2,2),(3,3),\\(4,4),(4,5),(5,4),(5,5),(6,6)\end{matrix}\right\}$ find the partition of A induced by R.	8	K4	CO3
		ii	Show that every chain is a distributive lattice.	8	K4	CO3

(OR)

	b)	i	If R is the relation on $A = \{1, 2, 3\}$ represented by the matrix $M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ find the matrix representing $R^{-1}, \bar{R}$ and R^2 and also express them as ordered pairs.	8	K4	CO3
		ii	Draw the Hasse diagram representing the partial ordering $\{(A, B)/A \subseteq B\}$on the power set P(S) where $S = \{a, b, c\}$. Find the maximal, minimal, greatest and least elements of this partially ordered set. Is it complemented Lattice? Justify your answer.	8	K4	CO3
14.	a)	i	If $A = \{1, 2, 3, 4, 5\}, B = \{1, 2, 3, 8, 9\}$ and the functions $f: A \to B$ and $g: A \to A$ are defined by $f = \{(1, 8), (3, 9), (4, 3), (2, 1), (5, 2)\}$ and $g = \{(1, 2), (3, 1), (2, 2), (4, 3), (5, 2)\}$, find $f \circ g$, $g \circ f, f \circ f$ and $g \circ g$ if they exist.	8	K5	CO4
		ii	Using characteristic function show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ where A,B and C are subsets of E.	8	K2	CO4
			(OR)			
	b)	i	If $f: Z \to N$ is defined by $f(x) = \begin{cases} 2x - 1, & if\ x > 0 \\ -2x, & if\ x \leq 0 \end{cases}$. Prove that f is one- to- one and onto, also find f^{-1}.	8	K5	CO4
		ii	Solve the recurrence relation $a_n = a_{n-1} + 1$ with critical term $a_o = 4$.	8	K2	CO4
15.	a)	i	If $f: G \to G'$ be a group homomorphism and H is a subgroup of G', then $f^{-1}(H)$ is a subgroup of G	8	K3	CO5
		ii	Find the left cosets of $\{[0], [3]\}$ in the addition modulo group $(Z_6, +_6)$.	8	K3	CO5
			(OR)			
	b)	i	Prove that the order of a subgroup of a finite group is a divisor of the order of the group	8	K3	CO5
		ii	Prove that the set $Z_4 = \{0,1,2,3\}$ is a commutative ring with respect to the binary operation $+_4$ *and* $\times_4$.	8	K3	CO5